

**ONE-HUNDRED PROBLEMS
WITH AN
ANSWER OF 100**

G. Patrick Vennebush

The Story of this Problem Collection

For the 2020 Annual Meeting of the National Council of Teachers of Mathematics, I submitted a proposal titled, “One-Hundred Problems Involving the Number 100,” that, as the name implies, outlined a session during which I planned to share 100 problems. The conference was meant to be a celebration of NCTM’s 100 years of service to mathematics education, and my session was meant to be a celebration of NCTM.

My proposal was accepted, and I relied on the collective wisdom of hundreds of educators, thousands of resources, and more hours than I care to count to prepare a collection of 100 interesting problems. Alas, it seemed all for naught, because the 2020 Annual Meeting — scheduled for early April — was cancelled in late March when the pandemic hit.

But there’s a silver lining. As part of NCTM’s 100 Days of Professional Development, [One-Hundred Problems](#) was delivered as a webinar on May 14, 2020. The session was a rousing success, with over 800 attendees appreciating the opportunity to laugh, connect, and solve problems. Afterwards, several members of the NCTM staff and NCTM President Trena Wilkerson stayed on the line to debrief and celebrate. Jokingly, Trena suggested, “Now we need a collection of 100 problems for which the answer is 100.”

And, really, that’s it.

That suggestion led me to write the 100 problems you’ll find on the following pages. If you love them, by all means, let me know. And if you hate them, please email Trena and let her know that this was a terrible idea.

The problems on the following pages are roughly divided into topics — although really great problems often defy categorization, so take it with a grain of salt — and the problems mostly proceed from easy to difficult, though such distinctions are highly subjective and, often, personal. For maximum enjoyment, my recommendation is to flip between sections and jump from problem to problem quite often; only stop and sink your teeth into those that pique your enthusiasm.

But, seriously, I don’t care how you use this collection. The truth is, if you use it at all, I’ll be honored.

Enjoy!

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Post-script: The book [One-Hundred Problems Involving the Number 100](#) was published by NCTM and released on November 11, 2020.

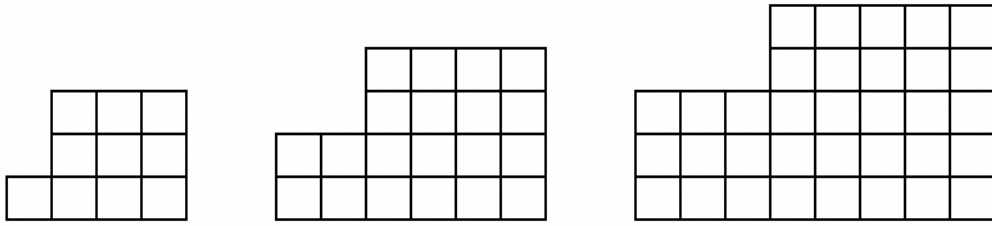
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PATTERNS and ARITHMETIC

Problem 1

If the pattern below continues, how many squares would be in the sixth figure?



Problem 2

A caterpillar works hard to climb 9 cm up a pole every day, only to slide down 5 cm every night. How many days will it take the caterpillar to reach the top of the pole, which is 405 cm above the caterpillar?

Problem 3

Together, a bat and a baseball cost \$110. The ball costs \$90 less than the bat. How much is the bat?

Problem 4

What is the value of the following expression?

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

Problem 5

What is $(1 + 2 + 3 + 4)^2$?

Problem 6

What is $100 + 40 - 100 + 30 + 100 + 20 - 100 + 10$?

Problem 7

There are 10 dog owners in the Portland Dog Walking Club, and each owner has 2 dogs. How many legs are in the club?

Problem 8

What comes next?

1, 4, 9, 16, 25, 36, 49, 64, 81, ___

Problem 9

What comes next?

1, 12, 23, 34, 45, 56, 67, 78, 89, __

Problem 10

What comes next?

1, 2, 10, 20, ___

Problem 11

What comes next?

0, 1, 10, 11, ___

Problem 12

Two numbers on a hundreds chart, one directly above the other, have a product of 2,475. What is the sum of the numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Problem 13

What's the value of the following expression?

$$0.001 \times \frac{1}{100} \times 10^7$$

Problem 14

What is the value of $6 \times 8 \div 2 + (20 - 1) \times 4$?

Problem 15

What is the value of $26^2 - 24^2$?

Problem 16

What is the value of $7^3 - 3^5$?

Problem 17

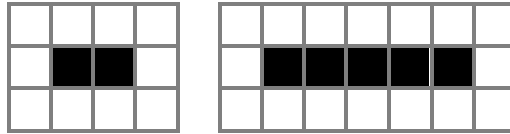
What is $5 \times 6! \div 6^2$?

Problem 18

What is the value of $20 \div 5(20 + 5)$?

Problem 19

The first figure below shows 2 black squares surrounded by 10 white squares; the second figure shows 5 black squares surrounded by 16 white squares. How many white squares would surround a strip of 47 black squares?

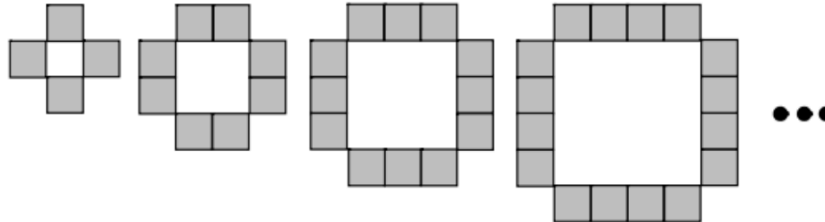


Problem 20

One-hundred three people in a circle are given the numbers 1 to 103, in order. Going clockwise around the circle starting with the person holding number 1, every third person leaves the circle. The first to leave is 3, the next to leave is 6, and so on. As some leave, those remaining form a smaller circle without gaps. People continue to leave in this manner until only one person remains. Who will be the last one remaining?

Problem 21

In each figure below, gray unit squares form the border around a white square. In Stage 1, for instance, the gray area is four square units and the white area is one square unit. In Stage n , the ratio of the area of the white interior square to the combined area of the gray squares is 25. What is the value of n ?



NUMBER THEORY

Problem 22

What is the least positive integer that is divisible by 4, 10, and 25?

Problem 23

What is the 14th term of the sequence 9, 16, 23, ...?

Problem 24

In the sequence of positive integers from 1 to 162, how many times does the digit 1 appear?

Problem 25

It takes 192 digits to number the pages of a book, starting at 1. How many pages does the book have?

Problem 26

What is the sum of the first nine prime numbers?

Problem 27

If two positive integers have a sum of 20, what is their maximum possible product?

Problem 28

What is $1 + 3 + 5 + \cdots + 19$?

Problem 29

The number of lily pads on a pond doubles every day. There were a few lily pads on Day 1. On Day 102, the entire pond was covered by lily pads. On which day was a quarter of the pond covered?

Problem 30

When $407!$ is expanded, how many trailing zeroes will there be?

Problem 31

A positive integer has the following properties:

- The sum of its digits is 1.
- The product of its digits is 0.
- It has nine positive divisors.

What is the number?

Problem 32

What is $100_5 - 100_4$, written in base 3?

Problem 33

From a set of the first n positive integers, Lonnie found the sum of $n - 1$ of them. The result was 20,000. Which integer was not included in the sum?

Problem 34

How many positive integer divisors does 210,000 have?

Problem 35

A partition of a positive integer is a way to write that integer as a sum of one or more smaller positive integers. For instance, the five partitions of 4 are $1 + 1 + 1 + 1$, $1 + 1 + 2$, $1 + 3$, $2 + 2$, and 4. What is the difference between the number of partitions of 13 and the number of partitions of 1?

Problem 36

What is the smallest number that leaves remainder 1 when divided by 3, 2 when divided by 7, and 4 when divided by 8?

Problem 37

In the multiplication table below, the numbers 2, 3, 4, 5, and 6 are used to replace a , b , c , d , and e . The six products within the multiplication table are then computed, and the sum of the six products is found. What is the maximum possible value for that sum?

\times	a	b	c
d			
e			

Problem 38

In the land of Honah Lee, coins are available in denominations of 7, 19, and 107 zlorumbis. What is the greatest number of zlorumbis that cannot be obtained using some combination of these coins?

ALGEBRA and FUNCTIONS

Problem 39

The difference between two numbers is 2, and the difference between their squares is 200. What is the sum of the two numbers?

Problem 40

$$\text{😊} \times \text{😊} = 25$$

$$\text{😊} \times \text{😐} \times \text{😐} = 20$$

$$\text{😐} \times \text{😐} \times \text{😞} = 40$$

$$\text{😊} \times \text{😐} \times \text{😞} = ?$$

Problem 41

Stacy is $\frac{4}{5}$ as old as her mom. Twenty years ago, Stacy was $\frac{3}{4}$ as old as her mom. How old is Stacy's mom?

Problem 42

A basset hound weighs 60 pounds plus two-fifths of its own weight. How much does the basset hound weigh?

Problem 43

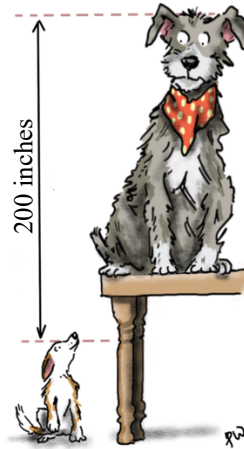
The sum of two numbers is 29. The difference of the same two numbers is 21. What is the product of those two numbers?

Problem 44

The points $(-5, a)$ and $(20, b)$ lie on the parabola $y = x^2$. When those two points are connected by a straight line, what is the value of the y -intercept?

Problem 45

When a large dog sits on the ground and a small dog sits on a table, their heads are at the same height. When the small dog sits on the ground and the large dog sits on the table, the head of the small dog is 200 inches below the head of the large dog. In inches, what is the height of the table?

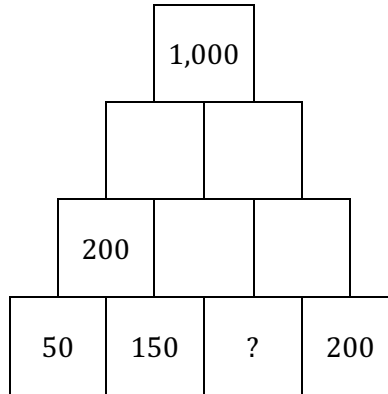


Problem 46

For what value of n does $0 - 1 + 2 - 3 + 4 - 5 + \cdots + (n - 2) - (n - 1) + n = 50$?

Problem 47

Above the bottom row, each number in a square is the sum of the two numbers below it.
What value should replace the question mark?



Problem 48

Alex was going fishing. He averaged 150 kilometers per hour on his drive to the lake. He took the same route on the way back, but he got a flat tire, so he only averaged 75 kilometers per hour on the return trip. In kilometers per hour, what was his average speed for the entire trip?

Problem 49

Jason's dad asked him to mow the grass once a week last summer. For mowing the grass 15 weeks, his dad promised to pay him \$100 and buy him a new bike. But Jason didn't cut the grass for 3 weeks because he went to the beach, so his dad bought him a new bike and paid him \$60. If this was a fair payment for the work that Jason did, how much is the bike worth, in dollars?

Problem 50

If $a + b = \sqrt{132}$ and $ab = 16$, what is the value of $a^2 + b^2$?

Problem 51

If $1/a + 1/b = 1/10$ and $a + b = 10$, what is ab ?

Problem 52

If x and y are both integers, how many different solutions exist for $|x| + |y| = 25$?

Problem 53

For the equation below, what is the least possible value of y ?

$$y = |||x - 100| + 100| - 100| + 100$$

Problem 54

Find the sum of all solutions to the following equation.

$$(x - 36)^{x^2 - 28x + 75} = 1$$

MEASUREMENT

Problem 55

Trijntje Keever, reportedly 8 feet 4 inches tall, is believed to be the tallest woman that ever lived. What was her height in inches?

Problem 56

Before Cruella Deville adopted 50 labradoodles from the rescue, 99% of the dogs at the rescue were labradoodles. After she adopted them, 98% of the remaining dogs were labradoodles. How many dogs were at the rescue before Cruella adopted any?

Problem 57

Haley has a bag containing \$100 in dimes. Harry has a bag containing only quarters, and the weight of Harry's bag is the same as the weight of Haley's bag. In dollars, what is the total value of the quarters in Harry's bag?

Problem 58

Haley has a bag containing 40 pennies. Harry has a bag containing 40 nickels. In grams, how much more does Harry's bag weigh?

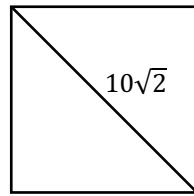
Problem 59

A stick measures 400 centimeters and is to be cut into two pieces. If the cut will be made randomly at any point along the stick's length, what is the average length of the smaller piece, in centimeters?

GEOMETRY

Problem 60

What is the area of a square with diagonal shown below?

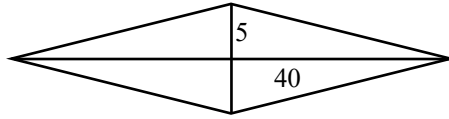


Problem 61

Two sides of a triangle have lengths of 8 cm and 25 cm. In square centimeters, what is the greatest possible area of the triangle?

Problem 62

What is the area of the rhombus shown below, with diagonals of 5 units and 40 units?



Problem 63

A rectangular prism with integer edge lengths has a volume of 56 cubic centimeters. What is the least possible surface area of the prism?

Problem 64

Sixty-six unit squares are used to form a rectangle. What is the difference between the greatest and least possible perimeters?

Problem 65

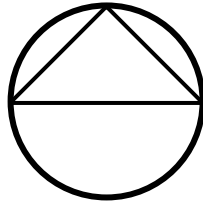
The faces of a rectangular prism have areas of 10, 20, and 50 square inches. In cubic inches, what is the volume of the solid?

Problem 66

In square inches, what is the maximum possible area of a rectangle whose perimeter is 40 inches?

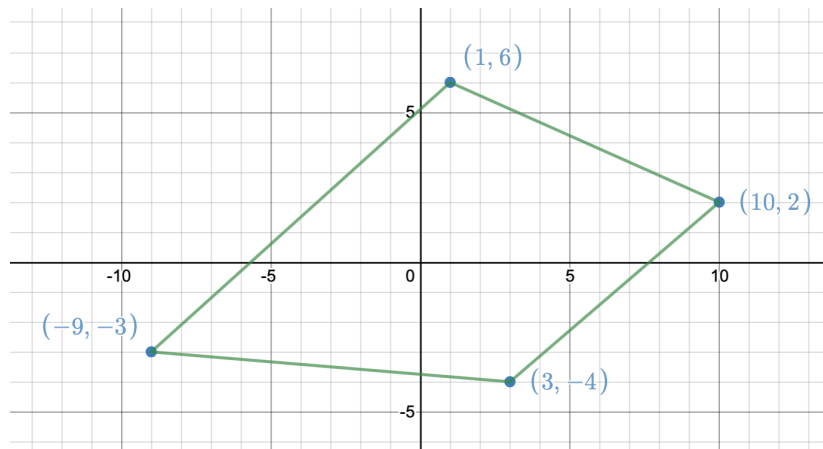
Problem 67

In the figure below, the longest side of an isosceles triangle is also the diameter of a circle of radius 10 cm. What is the number of square centimeters in the area of the triangle?



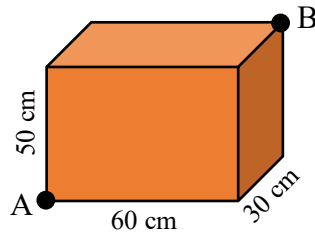
Problem 68

What is the area of the quadrilateral shown?



Problem 69

An ant crawls along the surface of a right rectangular prism that measures $60\text{ cm} \times 50\text{ cm} \times 30\text{ cm}$. In centimeters, what is the shortest distance the ant can travel to get from A to B (opposite corners)?



Problem 70

How many different cubes of any size with integer edge lengths are contained within a $4 \times 4 \times 4$ cube?

Problem 71

How many different rectangles with integer side lengths have a perimeter of 400 inches?

Problem 72

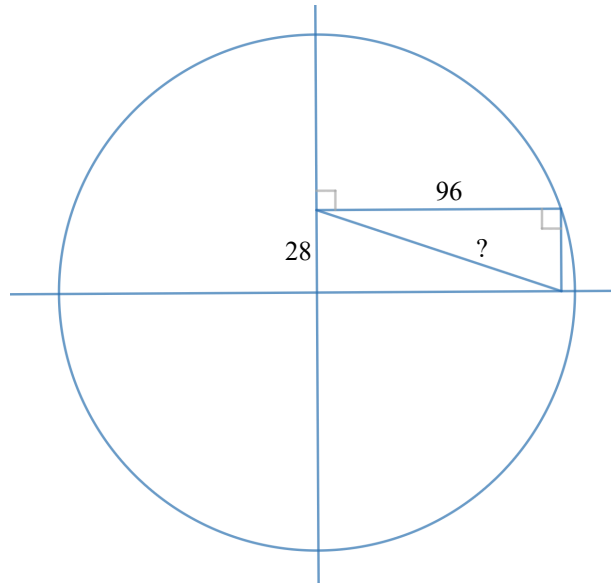
What is the hypotenuse of a right triangle with legs that are 60 units and 80 units long?

Problem 73

A large polyhedron has 50 faces and 52 vertices. How many edges does it have?

Problem 74

One vertex of the rectangle lies at the origin, the opposite vertex lies on the circle, and two of its sides coincide with the axes. The dimensions of the rectangle are $28\text{ cm} \times 96\text{ cm}$. In centimeters, what is the length of the diagonal shown?



Problem 75

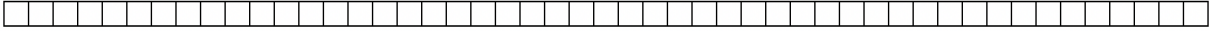
Two chords of a semicircle, measuring 60 cm and 80 cm, respectively, are parallel to the base of a semicircle. The distance between the chords is 10 cm. In centimeters, what is the diameter of the semicircle?

Problem 76

The length and width of a pool are both an integer number of feet, and the length is three times the width. A border of 1-foot-by-1-foot marble tiles is placed around the border of the pool. If the area of the pool is 432 square feet, how many tiles are needed for the border?

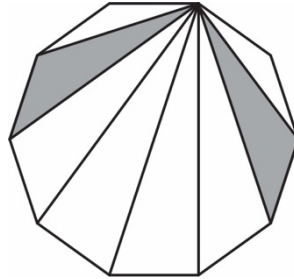
Problem 77

The figure below consists of 49 unit squares, and adjacent squares share a side. What is the perimeter of the figure?



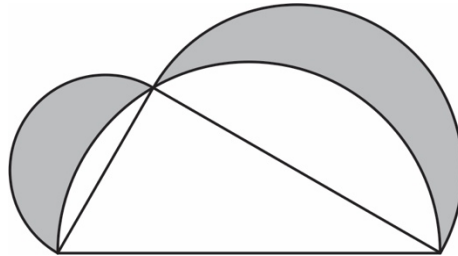
Problem 78

In the regular decagon shown below, the area of each shaded region is 10 square units. In square units, what is the area of the decagon?



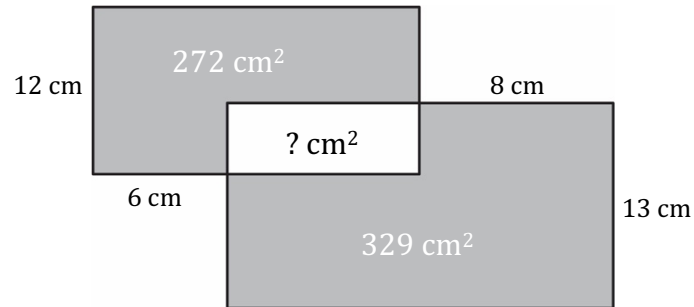
Problem 79

In the figure below, a right triangle is inscribed in a semicircle, the hypotenuse of the triangle is the diameter of the semicircle, and a semicircle is drawn on each leg of the triangle. The area of the triangle is 100 square units. What is the combined area of the two shaded regions, in square units?



Problem 80

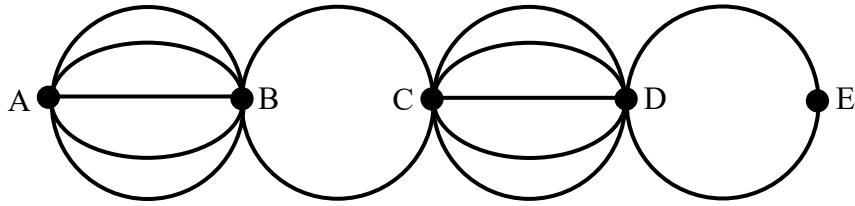
The shape below consists of two rectangles. The segments and regions have the lengths and areas indicated. What is the missing area?



COMBINATORICS

Problem 81

How many paths from A to E pass through B, C, and D exactly once each?



Problem 82

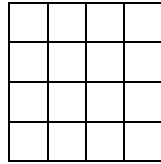
How many five-digit numbers containing the digits 1, 2, 3, 4, and 5 are greater than 15,300?

Problem 83

How many three-digit numbers are divisible by 5 and have an odd hundreds digit?

Problem 84

How many rectangles of any size are contained in the 4×4 grid shown below?



Problem 85

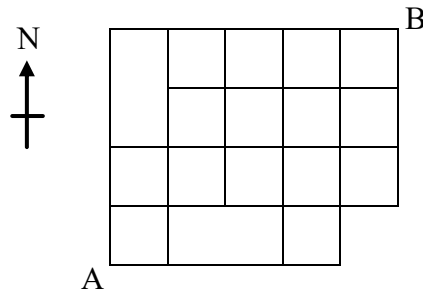
Depending on the event, the chairs in the social hall can be arranged in rows with 10, 20, or 25 chairs each, and in each case, there will be no chairs left over. What is the least number of chairs that could be in the social hall?

Problem 86

A Little League roster in Springfield consists of ten kids. Nine kids from the Jaguars roster are randomly chosen to play defense, and one kid from the Panthers roster is randomly chosen to bat. How many ways are there to choose the players?

Problem 87

Moving only north or east on the segments of the diagram, how many distinct paths are possible from A to B?



Problem 88

Starting at the M in the center square and moving to adjacent squares horizontally, vertically, or diagonally, how many paths will spell MATH in four moves?

		H	H	H	H	H	
H	T	T	T	T	T	H	
H	T	A	A	A	T	H	
H	T	A	M	A	T	H	
H	T	A	A	A	T	H	
H	T	T	T	T	T	H	
		H	H	H	H	H	

Problem 89

Customers at Ellie's Deli create a super sub by choosing three of the six meats that are available and selecting one of the five types of bread. How many different super sub combinations are possible?

Problem 90

A carnival game offers 6 large, 5 medium, and 9 small prizes to winners. If you get all five balls into the basket, you can choose one large, two medium, or three small prizes. How many different prize combinations are there?

STATISTICS

Problem 91

What is the mean of the first 199 positive integers?

Problem 92

What is the mean of the first 100 odd natural numbers?

Problem 93

On four tests, Eris scored 96, 82, 94, and 78. What score does she need to get on the fifth test to have an average of 90?

Problem 94

The mean and median of a set of five positive integers are 34. What is the greatest possible value of an integer in this set?

LOGIC

Problem 95

Which of the following statements is true?

1. Exactly 1 of these statements is false.
2. Exactly 2 of these statements are false.
3. Exactly 3 of these statements are false.
- ⋮
99. Exactly 99 of these statements are false.
100. Exactly 100 of these statements are false.
101. Exactly 101 of these statements are false.

Problem 96

What is the minimum number of games needed to determine a champion in a double-elimination tournament with 51 teams?

Problem 97

If $BETA = 200$, $ZETA = 2,600$, and $THETA = 16,000$, what is the value of ETA ?

Problem 98

Choose a number.

- Add 10.
- Double the result.
- Subtract 10.
- Divide by 2.
- Add 5.
- Subtract your original number.
- Square the result.

What is the final value?

Problem 99

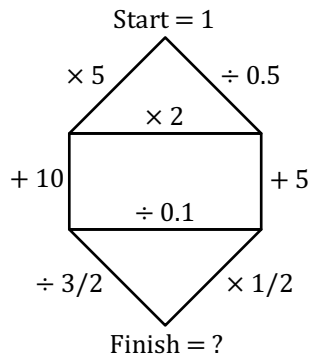
Four people have to cross an old, rickety bridge. It is in such bad shape, no more than two people can cross the bridge at the same time; and because it's dark, they must cross with a flashlight. Because the group has only one flashlight, when two people cross the bridge, one person must walk back with the flashlight for the next group. When two people cross the bridge, they travel at the speed of the slower person.

- Amir can cross the bridge in 5 minutes.
- Basiana can cross the bridge in 15 minutes.
- Cheslav can cross the bridge in 30 minutes.
- Dani can cross the bridge in 50 minutes.

What is the least amount of time for all four of them to cross the bridge?

Problem 100

Begin with a value of 1 at Start, and proceed along the paths to Finish. When a path is traveled, perform the indicated operation. If no path may be traveled more than once, what is the maximum possible value that can be obtained?



Problem 101

In a race, the person in last place is about to overtake the person who is currently in 99th place. How many people are in the race?

Answer Key

- | | | |
|---------|---------|----------|
| 1. 100 | 35. 100 | 69. 100 |
| 2. 100 | 36. 100 | 70. 100 |
| 3. 100 | 37. 100 | 71. 100 |
| 4. 100 | 38. 100 | 72. 100 |
| 5. 100 | 39. 100 | 73. 100 |
| 6. 100 | 40. 100 | 74. 100 |
| 7. 100 | 41. 100 | 75. 100 |
| 8. 100 | 42. 100 | 76. 100 |
| 9. 100 | 43. 100 | 77. 100 |
| 10. 100 | 44. 100 | 78. 100 |
| 11. 100 | 45. 100 | 79. 100 |
| 12. 100 | 46. 100 | 80. 100 |
| 13. 100 | 47. 100 | 81. 100 |
| 14. 100 | 48. 100 | 82. 100 |
| 15. 100 | 49. 100 | 83. 100 |
| 16. 100 | 50. 100 | 84. 100 |
| 17. 100 | 51. 100 | 85. 100 |
| 18. 100 | 52. 100 | 86. 100 |
| 19. 100 | 53. 100 | 87. 100 |
| 20. 100 | 54. 100 | 88. 100 |
| 21. 100 | 55. 100 | 89. 100 |
| 22. 100 | 56. 100 | 90. 100 |
| 23. 100 | 57. 100 | 91. 100 |
| 24. 100 | 58. 100 | 92. 100 |
| 25. 100 | 59. 100 | 93. 100 |
| 26. 100 | 60. 100 | 94. 100 |
| 27. 100 | 61. 100 | 95. 100 |
| 28. 100 | 62. 100 | 96. 100 |
| 29. 100 | 63. 100 | 97. 100 |
| 30. 100 | 64. 100 | 98. 100 |
| 31. 100 | 65. 100 | 99. 100 |
| 32. 100 | 66. 100 | 100. 100 |
| 33. 100 | 67. 100 | 101. 100 |
| 34. 100 | 68. 100 | |